淡江大學八十七學年度日間部轉學生入學考試試題

系別：統計學系三年級　　　　科目：機率與微積分

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1) Suppose that the distribution function of the random variable X is given by

$$F_X(x)=\begin{cases}0, & x<0\\ \frac{x}{4}, & 0\le x<1\\ \frac{1}{2}+\frac{x-1}{4}, & 1\le x<2\\ \frac{11}{12}, & 2\le x<3\\ 1, & 3\le x\end{cases}$$

(a) Find $P(X=x)$, $x=1,2,3$. (6%)

(b) Find $P\left(\frac{1}{3}<X<\frac{3}{2}\right)$. (3%)

(c) Is X discrete? continuous? or neither? Why? (3%)

2) Let X, Y be two random variables taking on the values -1, 0, 1 with the following respective probabilities:

$$f_{X,Y}(-1,-1)=\alpha,\quad f_{X,Y}(-1,0)=\beta,\quad f_{X,Y}(-1,1)=\alpha$$
$$f_{X,Y}(0,-1)=\beta,\quad f_{X,Y}(0,0)=0,\quad f_{X,Y}(0,1)=\beta$$
$$f_{X,Y}(1,-1)=\alpha,\quad f_{X,Y}(1,0)=\beta,\quad f_{X,Y}(1,1)=\alpha$$

$$\alpha,\beta>0,\quad \alpha+\beta=\frac{1}{4}.$$

(a) Find the means and the variances of X, Y, respectively. (8%)

(b) Find $Var(2X+Y+3)$. (6%)

3) Let Z be a standard normal random variable. Show that for $x>0$,

(a) $P(Z>x)=P(Z<-x)$. (5%)

(b) $P(|Z|>x)=2\,P(Z>x)$. (5%)

4) Let X and Y be two random variables such that their variances $Var(X)>0$ and $Var(Y)>0$.

(a) Prove that $-1\le\rho(X,Y)\le 1$., where $\rho(X,Y)$ is the correlation coefficient of X and Y. (8%)

(b) Prove that if $Y=a+bX$, $b>0$, then $\rho(X,Y)=1$. (6%)

5) (a) Is there any value of k that will make

$$f(x)=\begin{cases}\dfrac{\sin x}{2x}, & x\neq 0\\ k, & x=0\end{cases}$$

continuous at $x=0$? If so, what is it. Given reasons for your answer. (4%)

(b) State the Mean Value Theorem. (5%)

(c) Let f be a function continuous on $[a,b]$ and differentiable on (a,b). Prove that if $f'(x)>0, \forall x\in(a,b)$, then f increases on $[a,b]$. (5%)

6) Find the following limits: (12%)

(a) $\displaystyle\lim_{x\to\infty}\frac{2x^{\frac{5}{3}}-x^{\frac{1}{3}}+7}{x^{\frac{8}{5}}+3x+\sqrt{x}}$

(b) $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$

(c) $\displaystyle\lim_{x\to\infty}\frac{1}{x\ln x}\int_1^x \ln t\, dt$

7) (a) Let f be a continuous function. Find the value of the integral

$$I=\int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx$$

by making the substitution $y=a-x$ and adding the resulting integral to I. (6%)

(b) Find the value of the integral $\displaystyle\int_0^\infty x\,e^{\frac{-3x^2}{2}}\,dx$. (4%)

8) (a) Show that $\displaystyle\int_{-a}^{a} f(x)\,dx=\begin{cases}2\displaystyle\int_0^a f(x)\,dx, & \text{if } f \text{ is even}\\ 0, & \text{if } f \text{ is odd}\end{cases}$ (8%)

(b) Show that $\displaystyle\int_0^x\left(\int_0^u f(t)\,dt\right)du=\int_0^x f(u)(x-u)\,du$. (6%)