

# 淡江大學 103 學年度日間部轉學生招生考試試題

系別：產業經濟學系三年級

科目：微 積 分

考試日期：7 月 20 日(星期日) 第 4 節

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1. ( 18% ) Evaluate the limit if it exists; explain the reason if it doesn't exist.

(a)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$ ,

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ ,

(c)  $\lim_{x \rightarrow 0^+} x^{\sin(2x)}$ .

2. ( 32% ) Evaluate the integrals

(a)  $\iint_{\mathbb{R}} x^3 y^2 dA$ , where  $A = \{(x, y) \in \mathbb{R}^2 | 0 \leq x \leq 1, 2 \leq x \leq 4\}$ .

(b)  $\int_0^{\infty} 5x^2 e^{-x^3} dx$ .

(c)  $\int_0^1 \int_{\sqrt{x}}^1 x e^{5y^5} dy dx$ .

(d)  $\iint_{\Omega} \frac{\sin x}{x} dA$ , where  $\Omega$  is the region enclosed by the line  $y = 0$ ,  $x = 1$  and  $y = x$ .

3. (10%) The weekly demand for the digital camera type  $M$  is given by the demand function

$$p(x) = -0.025x + 300, \quad 0 \leq x \leq 12000,$$

where  $p$  denotes the unit price in dollars and  $x$  denotes the quantity demanded. The corresponding weekly total cost function is

$$C(x) = 0.000001x^3 - 0.015x^2 + 200x + 40000.$$

Find the level of production that will yield a maximum profit for the manufacturer.

4. (10%) Find the volume of the prism whose base is the triangle in the  $xy$ -plane bounded by the  $x$ -axis and the line  $y = x$  and  $x = 1$  and whose top lies in the plane  $z = 3 - x - y$ .

5. ( 10% ) Find the tangent plane to the surface

(a)  $z = x^2 - y^2$  at  $(2, 1, 3)$ .

(b)  $z^2 + 2z + x^2 + 2x + y^2 + 2y = 6$  at  $(0, 0, 2)$ .

6. (10%) Find the absolute maximum and minimum values of  $f(x, y) = x^2 + y^2 - 2x$  on the closed triangular region with vertices  $(2, 0)$ ,  $(0, 2)$  and  $(0, -2)$ .

7. (10%) Find the maximum and minimum values of  $f(x, y) = \sqrt{x^2 + y^2}$  subject to the constraint  $x^4 + y^4 = 32$ .