淡江大學 104 學年度日間部轉學生招生考試試題

系別：資訊工程學系三年級　　科目：離 散 數 學

考試日期:7月26日(星期日) 第3節　　本試題共 6 大題， 1 頁

1. Let $A = \{a, b\}$, $B = \{1, 2, 3\}$, and $C = \{x, y\}$. (20%)
   (a) Find $A \times B \times C$.
   (b) Give a general formula to find $| A \times B \times C |$.

2. Prove that every integer whose square is a multiple of 3 must divide by 3; i.e.,
   $\forall x \, \exists k \, \exists m \, ((x^2 = 3k) \rightarrow (x = 3m))$, where x, k, and m are integers. (20%)
   (Hint: prove by contraposition is must easier.)

3. Find the solution to the following recurrence relation and initial condition:
   $a_n = a_{n-1} + n + 1$, $a_0 = 1$. (20%)

4. (a) Find a recurrence relation for the number of bit strings of length n that contains two consecutive 0s. (10%)
   (b) What are the initial conditions? (5%)
   (c) How many bit strings of length ten contain two consecutive 0s. Use the above recurrence relation in (a) and (b) to solve it. (5%)

5. Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c, and d are constants. Determine necessary and sufficient conditions on the constants a , b, c, and d so that $f \circ g = g \circ f$. (10%)

6. Let $R = \{(1, 1), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4)\}$. (10%)
   (a) Represent the relation R by a matrix. (4%)
   (b) Draw a directed graph to represent R. (4%)
   (c) Show what properties are satisfied by R. (2%)