

淡江大學 103 學年度日間部轉學生招生考試試題

系別：資訊工程學系三年級

科目：離散數學

考試日期：7月20日(星期日) 第3節

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1. Multiple Choice (單選題 3pt@8=24pts)

(a) Find a relationship to best describe the relation for the pair of sets: $A \cup B$, $A \cup (B - A)$.

① \subseteq ② $=$ ③ \supseteq ④ None of above.

(b) Define a function $f: \mathbb{N} \rightarrow \mathbb{N}$ such that $f(n) = 4n^2 + 1$. Then, f is one to one. ① True ② False.

(\mathbb{N} is the set of natural numbers)

(c) The negation of "Some people like pizza." is ① Some people like spaghetti. ② Some people do not like pizza. ③ Everyone likes pizza. ④ Everyone does not like pizza. ⑤ None of above.

(d) Let $A = \{a, \{a\}, \{a, \{a\}\}\}$. Then $|A| =$ ① 1 ② 2 ③ 3 ④ 4 ⑤ None of above.

(e) Consider a rule for generating the terms of the sequence that begins 1, 3, 4, 8, 15, 27, 50, 92, ..., find the next term of the sequence. ① 144 ② 169 ③ 196 ④ 225 ⑤ None of above.

(f) The coefficient of x^2y in $(x + 2y - 5)^4$ is ① 10 ② -10 ③ 24 ④ -24 ⑤ None of above.

(g) The number of non-negative integer solutions in $x + y + z = 8$ is ① 45 ② 85 ③ 125 ④ 165

⑤ None of above.

(h) Let $\sum_{i=0}^2 \sum_{j=0}^2 (3i + 2^j) =$ ① 16 ② 24 ③ 40 ④ 48 ⑤ None of above.

**** Answer the problems 2 ~ 6 in details to get full credits.**

2. Determine whether $[\neg p \wedge (p \leftrightarrow q)] \rightarrow \neg q$ is a tautology by constructing a truth table. (10 pts)

3. Prove by **induction**: $1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + n \cdot 2^n = (n-1)2^{n+1} + 2$ for all positive integer n . (10 pts)

4. Define $S = \{a, b, c, d, e\}$. (a) How many permutations are there to arrange $a \sim e$ in a row such that a and e are **not together**; (b) find the number of the **onto** functions f from S to $\{1, 2, 3\}$ such that $f(a) = 1$. (20 pts)

5. Define $A = \{-7, -6, -5, \dots, 6, 7\}$ and a relation R on A such that $(a, b) \in R \Leftrightarrow a \equiv b \pmod{4}$ (10 + 6 pts)

(a) Show the relation R is an equivalence relation; (b) indicate the corresponding partition on A .

6. Consider the graph on the right, answer the following.

Indicate the details of every step of the algorithm. (20 pts)

(a) Use Dijkstra's algorithm to find the length a shortest path between the vertices a and i . (必須標示出每一個步驟的結果)

(b) Use Kruskal's algorithm to find a minimum spanning tree.

(必須標示出邊被選擇的順序)

