5-1

淡江大學 102 學年度日間部轉學生招生考試試題

系別：資訊工程學系三年級　　科目：離散數學

考試日期：7 月 24 日(星期三) 第 4 節　　本試題共 8 大題，1 頁

1. Use a ***proof by contradiction*** to prove: (10%)

   For all integers n, if n^2 is odd, then n is odd.

2. Using as few groups as possible, put the words fish, sit, stay, play, diet, tree, root, duck, dog, and hen into groups such that none of the words in a group have letters in common. Use a graph model and graph coloring. How many groups are used? What are your groups? Justify your answer: explain why your grouping uses the fewest groups possible. (15%)

3. Let X be a finite set. For subsets $A, B \in P(X)$, let $A\ R\ B$ if $|A| = |B|$. This is equivalence relation on $P(X)$. If $X = \{1, 2, 3, 4, 5\}$, list the equivalence classes. (10%)

4. Let $F \subseteq \mathbf{N}$ be the set of all factors of 270. (20%)
   (a) List all elements of set F. ($F = \{ ? \}$)
   (b) How many elements in F? ($|F| = ?$)
   (c) Draw the Hasse diagram for $(F, |)$.
   (d) List all minimal and maximal elements.

5. The complete graph on n vertices (denoted K_n) is the undirected graph with exactly one edge between every pair of distinct vertices. (15%)
   (a) Derive a formula for the number of edges in K_n.
   (b) Does there exist an Euler circuit in K_n? Justify your answer.
   (c) Does there exist a Hamilton circuit in K_n? Justify your answer.

6. For the following sequence: (10%)

   5, 11, 18, 26, 35, 45, …

   (a) find the recurrence relation.
   (b) find the closed-form solution.

7. Find the solution to the linear homogeneous recurrence relation (10%)

   $$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$$

   with the initial conditions $a_0 = 2$, $a_1 = 5$, and $a_2 = 15$.

8. An urn contains 6 red balls and 8 blue balls. Four balls are drawn at random, without replacement. (10%)
   (a) What is the probability that all four balls are red?
   (b) What is the probability that two of the balls are red and two are blue?