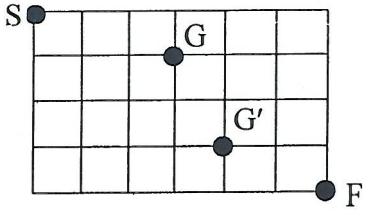
淡江大學 100 學年度轉學生招生考試試題

系別：資訊工程學系三年級　　科目：離 散 數 學

考試日期：7 月 19 日(星期二) 第 4 節　　本試題共 12 大題，　1 頁

1. Show: $\forall x \in \mathbf{R}$, if $x^2 \neq 1$, then $x \neq 1$. (10%)
2. Use mathematical induction to prove the statement: $\forall n \geq 3$, $n^2 \geq 2n + 1$. (10%)
3. Find (a) gcd(259, 77) and (b) write it in the form $259x + 77y$ for some $x, y \in \mathbf{Z}$. (10%)
4. Determine the smallest element of the given set $S = \{s : s = 3n + 20$, where $n \in \mathbf{Z}$ and $3n + 20 > 0\}$. (10%)
5. Make a Hasse diagram for the relation "divides" relation | on {2, 3, 5, 6, 10, 12}. (10%)
6. Partition the set of numbers {20, 500, 5, 176, 80, 605} according to the equivalence relation: "has the same set of prime divisors as." (10%)
7. Find the coefficient of $x^{493}y^7$ in $(x - 2y)^{500}$. (5%)
8. Compute 13^{200} mod 7. (5%)
9. A certain ATM code is required to consist of 4 distinct digits (0 to 9). If the ATM machine will allow only 3 guesses per day, then how many days would it take to try all possible ATM codes? (5%)
10. If 4 balls are selected from a bag containing 6 red, 5 blue, and 4 green balls, then what is the probability of getting 2 red or an equal number of blue and green? (5%)
11. In the following figure, location S is a storehouse from which deliveries are made, and F is a factory to which a large is going. Intersections G and G′ contain gas stations to which deliveries are sometimes made and where the delivery truck can get gas if necessary. Since the factory is southeast of the storehouse, we are interested only in efficient delivery routes, which never travel north or west in this grid.
    (a) How many possible efficient delivery routes are there from the storehouse to the factory? It is not necessary to pass by a gas station. (5%)
    (b) Sometimes intermediate stops are required at both gas stations G and G′. (5%)



12. Let $V = \{1, 2, 3, 4, 5, 6\}$, $E = \{\{1, 4\}, \{2, 4\}, \{3, 4\}, \{3, 5\}, \{2, 3\}\}$, and $G = (V, E)$.
    (a) Draw $G = (V, E)$. (5%)
    (b) What is the distance from 1 to 5? Show the path. (5%)