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# 淡江大學 99 學年度轉學生招生考試試題

系別：資訊工程學系三年級　　　　科目：離　散　數　學

本試題共 7 大題， 1 頁

**1. Multiple Choice** (單選題) (25 pts)

_____(a) If k is the least numbers of integers must be selected from {1, 3, 5, …, 29} to make sure there is at least one pair of these integers with the sum 26, then k is (a) 6~8 (b)9~11 (c)12~14 (d) 14~16 (e) none of above.

_____(b) If k is the maximum height of *T* where *T* is a full binary tree with 101 vertices, then k is (a) 1~20 (b)21~40 (c) 41~60 (d) 61~80 (e) ≥ 81.

_____(c) If k is the smallest coefficient in the expansion of $(2x + 3)^5$ then k is (a) 1~20 (b)21~40 (c) 41~60 (d) 61~80 (e) ≥ 81.

_____(d) In 0, $2^n$, $n4^n$ and $(-4)^n$ these four numbers, the number of correct answers for the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ is (a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (所給的 4 個數字中是答案的有幾個)

_____(e) In $\phi \in \{\phi\}$, $\{\phi\} \subset \{\phi,\{\phi\}\}$, $\{\phi\} \in \{\phi,\{\phi\}\}$ and $\{\{\phi\}\} \subset \{\phi,\{\phi\}\}$ these four statements, the number of correct statements is (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

**2. True or False** (是非題) (10 pts)

_____(a) $\lfloor x \rfloor = \lceil \lfloor x \rfloor \rceil$ for all real numbers.

_____(b) $f: N \to N$ by $f(n) = n^2 + 1$ is an one-to-one function.

_____(c) There are no integer solutions x and y to the equation $2x^2 + 5y^2 = 14$.

_____(d) If the truth value for " p→ q" is **false** then the truth value of its **converse** " q→ p" must be **true**.

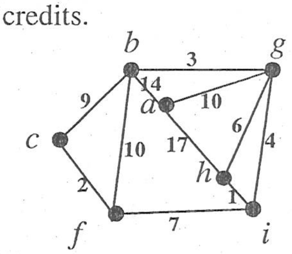
_____(e) The negation of " p→ q" is " ¬p→ ¬q".

**3.** Prove or disprove: If $a^2 \equiv b^2 \pmod 4$, where *a* and *b* are integers, then $a \equiv b \pmod 4$. (12 pts)

**4.** Find the smallest equivalence relation on {1,2,3} that contains (1,2). (12 pts) Justify your answer.

**5.** How many nonnegative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 21$ such that $x_1 \geq 5$ and $1 \leq x_2 \leq 8$ ? (12 pts) Show enough work to get full credits.

**6.** Apply Dijkstra's Algorithm to find a shortest path from ***a*** to ***f***.
Indicate what is your shortest path and the total weight of the path.
You must show every step in order to get full credits. (14 pts)




**7.** Use **mathematical induction** to prove that 3 divides $n^3+2n$ whenever $n$ is a nonnegative integer. (15 pts)
(3 整除 $n^3+2n$， $n$ 為非負整數) (必須以**歸納證明**的方法證得)