1. Provide a combinatorial argument to show that if n and k are positive integers with $n = 2k$, then $n!/2^k$ is an integer. (20%)

2. Solve the following recurrence relation: $2a_{n+2} - 11a_{n+1} + 5a_n, n \geq 0, a_0 = 2, a_1 = -8$. (20%)

3. Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$. (15%)

4. Let $a, b \in \mathbf{Z}$ so that $2a + 3b$ is a multiple of 17. Prove that 17 divides $9a + 5b$. (15%)

5. Find the coefficient of x^7 in $(1 + x + x^2 + x^3 + \ldots)^{15}$. (15%)

6. Find the generating function for the number of ways to select 10 candy bars from large supplies of six different kinds. (15%)