淡江大學九十一學年度日間部轉學生招生考試試題

系別：資訊工程學系三年級　　　科目：離 散 數 學

准帶項目請打「○」否則打「×」
計 算 機
X

● **題目 3 ～ 7 的所有答案必須將過程寫出。**

1. True or False (是非題) (4 pts each)

(a) $\wp(A \cup B) = \wp(A) \cup \wp(B)$. ($\wp(A)$ is the set containing all the subsets of A)

(b) There are 6 subsets of A that contains exactly 2 elements where A = {∅, {a}, b, {a,b} }.

(c) If e is an edge of a connected graph G and G − {e} is connected then G is impossible to be a tree.

(d) For a connected graph G, G has exactly one spanning tree.

(e) If $(q \wedge r) \rightarrow p$ and $q \rightarrow \neg r$, then p. (p, q, r are propositions.)

2. Multiple Choices (單選題) (4 pts each)

(a) Consider the generating function f(x) for the number of integer solutions for $x_1 + x_2 + x_3 + x_4 = 12, 0 \le x_i \le 6$ for $i = 1, 2, 3$, and $x_4 \ge 3$, then f(x) = ① $(1 + x + x^2 + ... + x^5 + x^6)^3 (x^3 + x^4 + x^5 + ... + x^{11} + x^{12})$ ②$(1 + x + x^2 + ... + x^5 + x^6 + ...)^3 (x^3 + x^4 + x^5 + ... + x^{11} + x^{12})$ ③ $(x + x^2 + ... + x^5 + x^6)^3 (x^3 + x^4 + x^5 + ... + x^{11} + x^{12} + x^{13} + ...)$ ④ $(x + x^2 + ... + x^5 + x^6)^3 (x^3 + x^4 + x^5 + ... + x^{11} + x^{12})$ ⑤以上皆非.

(b) The negation of $\forall x \forall y [(x < y) \rightarrow \exists z \{ x < z < y \}]$ is ① $\exists x \exists y [(x \ge y) \rightarrow \forall z \{(x \ge z) \vee (z \ge y)\}]$ ②$\exists x \exists y [(x < y) \rightarrow \forall z \{ (x \ge z) \vee (z \ge y)\}]$ ③$\exists x \exists y [(x \ge y) \rightarrow \forall z (x \ge z \ge y)]$ ④$\exists x \exists y [(x \ge y) \wedge (\forall z \{ (x \ge z) \vee (z \ge y)\})]$ ⑤$\exists x \exists y [(x < y) \wedge (\forall z \{(x \ge z) \vee (z \ge y)\})]$ ⑥以上皆非.

(for (c) ~ (e))　Consider the set of all 5-digit numbers S = { 10000, 10001, 10002, ..., 99999 }, and t = $8 \cdot 8 \cdot 7 \cdot 6 \cdot 5$. Let k_1 = number of odd integers(奇數)in S have no two digits the same(每個數字都不同), k_2 = number of even integers(偶數) in S have no two digits the same, k_3 = number of integers in S that have the sum of their digits to be 9 (數字總和為9,如 22212,10008,...等等),

(c) Which is correct? ① $k_1 > t$ ② $k_1 = t$ ③ $k_1 < t$.

(d) Which is correct? ① $k_2 > t$ ② $k_2 = t$ ③ $k_2 < t$.

(e) Which is correct? ① $k_3 > 500$ ② $k_3 = 500$ ③ $k_3 < 500$.

3. Find the coefficient of $x^2 y^3 z^3$ in $(\frac{x}{2} - 3y + 4z - 5)^{10}$. (只需列出計算式子，可以不算出最後的結果。) (10 pts)

4. Prove by induction: $1 + 2 + ... + n < \dfrac{(2n+1)^2}{8}$ for integer $n \ge 1$. (12 pts)

5. If A = { 1, 2, 3, 4 }, B = { a, b, c, d, e, f }, answer the following, (show your work) (14 pts)
(a) How many functions f: A --> B that f(1) = a ?
(b) How many **invertible** functions f : B --> B satisfying f(a) ≠ a ?

6. Is it possible to draw a loop-free connected undirected graph G = (V, E), where the degrees of the vertices are 1, 1, 2, 2, 2 ? How about 1, 1, 1, 2, 2? If yes, show the graphs; otherwise, explain why not. (12 pts)

7. If S = { 5, 8, 11, 14, ..., 56, 59}, how many elements must we select from S to ensure(保證)that there will be at least two whose sum is 70? Justify your answer to get the full credits. (12 pts)