淡江大學九十學年度日間部轉學生招生考試試題

Show enough work to justify your answers!!

1. Use mathematical induction to prove that the following statement is true

    $3n^5 + 5n^3 + 7n$ is divisible by 15 for each positive integer n.   (20%)

2. A man has 15 close friends of whom 6 are women.   (20%)

    (a). In how many ways can he invite 3 or more of his friends to a party?

    (b). In how many ways can he invite 3 or more of his friends if he wants the same number of men (including himself) as women?

3. Build a generating function for a_r = the number of integral solutions to the equation $e_1 + e_2 + e_3 = r$ if: (20%)

    (a). $0 \leq e_i \leq 3$ for each i.

    (b). $0 < e_i$ for each i.

4. Find an integer x where $0 \leq x \leq 12$ and $5^{243} \equiv x \mod 13$.   (20%)

5. Compute the adjacency matrix of the transitive closure of the relation R whose adjacency matrix is   (20%)

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$