

淡江大學八十九學年度日間部轉學生招生考試試題

系別：資訊工程學系三年級

科目：離散數學

53

本試題共 / 頁

Show enough steps to justify your answers.

Note that the symbol \rightarrow indicates “imply”, while \Rightarrow indicates “logical imply”.

1. If $\{x_1, x_2, \dots, x_7\} \subseteq \mathbb{Z}^+$, show that for some $i \neq j$, either $x_i + x_j$ or $x_i - x_j$ is divisible by 10. (10%)
2. Find and solve a recurrence relation for the number of binary sequences of length n that have no consecutive 1's (沒有相鄰的 1). (10%)
3. Prove or disprove: $\forall x (p(x) \vee q(x)) \Rightarrow \forall x p(x) \vee \forall x q(x)$ (10%)
4. Establish the validity of the following argument by Rule of Contradiction (反證法). (10%)
$$[(p \rightarrow (q \rightarrow r)) \wedge (p \vee s) \wedge (t \rightarrow q) \wedge \neg s] \rightarrow (\neg r \rightarrow \neg t)$$
5. In how many ways can one travel in the xy plane from $(1, 2)$ to $(8, 8)$ if each move is one of the following types: (10%)
(H): $(x, y) \rightarrow (x+1, y)$; (V): $(x, y) \rightarrow (x, y+1)$; (D): $(x, y) \rightarrow (x+1, y+1)$
6. On the X-Y plane, answer following questions with brief explanations. (15%)
 - (a) How to represent all points on the X-Y plane?
 - (b) Give a relation but not a function
 - (c) Give a function but not a relation
 - (d) Give a function but not an one-to-one function
 - (e) Give a function that is both one-to-one and onto function
7. For any universe U and any set $A, B \subseteq U$, prove the following statements are equivalent. DO NOT use the membership tables or Venn diagrams. (15%)
 - (a) $A \subseteq B$
 - (b) $A \cap B = \emptyset$
 - (c) $B \subseteq A$
8. Provide a recursive definition for the following language $A \subseteq \Sigma^*$ where $\Sigma = \{0, 1\}$ such that $x \in A$ if and only if the number of 0's in x is even. (10%)
9. How can Mary split up 12 hamburgers and 16 hotdogs among her sons Richard, Peter, Christopher, and James in such a way that James gets at least one hamburger and three hot dogs, and each of his brothers gets at least two hamburgers but at most five hot dogs? (10%)