

1. (20%) In a two-dimensional, planar flow of an incompressible fluid, the x-component of velocity is given by

$$u = x^2 + x - 2y$$

Can you find an expression for v ?

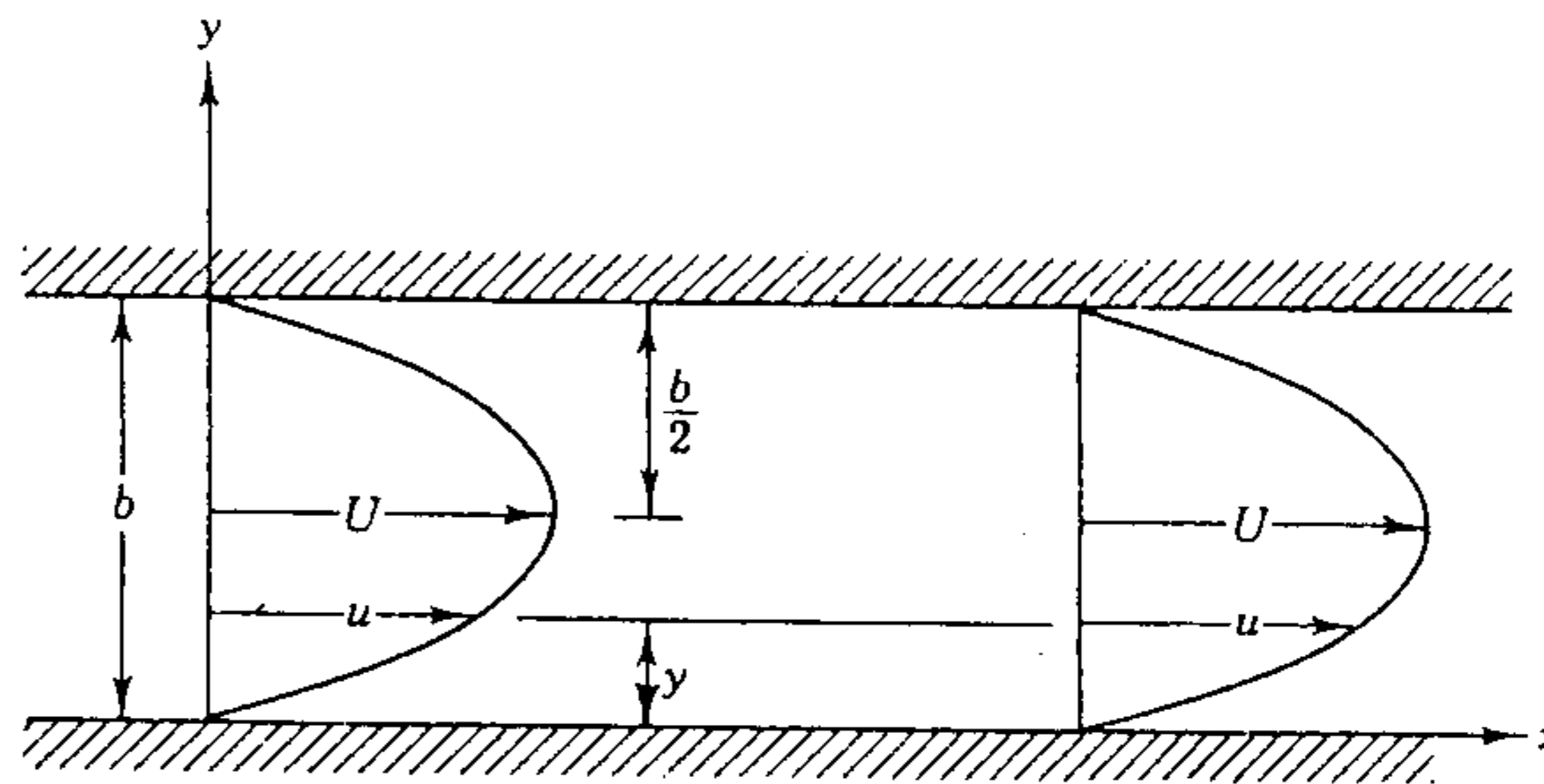
2. (20%) The velocity field of a steady, incompressible flow is given as

$$\mathbf{V} = (3x^2 - 2xy)\mathbf{i} + (y^2 - 6xy + 3yz^2)\mathbf{j} - (z^3 + xy^2)\mathbf{k}.$$

Determine the pressure gradient at the position $(x, y, z) = (2, 3, 1)$.

3. (20%) Calculate the circulation about the square enclosed by $x = \pm 1$ and $y = \pm 1$ in the xy -plane for the two-dimensional flow given by $\mathbf{V} = (3x^2 + y)\mathbf{i} - (6xy + x)\mathbf{j}$

4. (20%) A steady, two-dimensional, incompressible flow occurs between two fixed plane surfaces spaced at a distance b apart. The velocity profile is a parabola with its vertex at the center line. Determine the stream function for the flow. Is this flow irrotational?



5. (20%) For turbulent flow in a smooth pipe with $\rho V D / \mu < 10^5$, the velocity profile in the turbulent core can be approximated by the following power equation:

$$u = u_{\max} \left(\frac{y}{r_0} \right)^{1/7} = u_{\max} \left(\frac{r_0 - r}{r_0} \right)^{1/7}$$

where y and r are respectively the distances measured from the pipe wall and the pipe axis, and r_0 is the radius of the pipe. What is the average velocity V of this pipe flow if expressed in u_{\max} ?